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|---|---|---|---|---|
| **1** | (i)(a) | $1 - P(\le 6) = 1 - 0.8675$<br>$= \mathbf{0.1325}$ | M1<br>A1 **2** | 1 – .9361 or 1 – .8786 or 1 – .8558: M1. .9721: M0<br>Or 0.132 or 0.133 |
| | (b) | Po(0.42)<br>$e^{-0.42}\dfrac{0.42^2}{2!} = \mathbf{0.05795}$ | M1<br>M1<br>A1 **3** | Po(0.42) stated or implied<br>Correct formula, any numerical $\lambda$<br>Answer, art 0.058. Interpolation in tables: M1B2 |
| | (ii) | E.g. "Contagious so incidences do not occur independently", or "more cases in winter so not at constant average rate" | B2 **2** | Contextualised reason, referred to conditions: B2. No marks for mere learnt phrases or spurious reasons, e.g. not just "independently, singly and constant average rate". See notes. |
| **2** | (i) | B(10, 0.35)<br>P(< 3)<br>$= \mathbf{0.2616}$ | M1<br>M1<br>A1 **3** | B(10, 0.35) stated or implied<br>Tables used, e.g. 0.5138 or 0.3373, or formula ± 1 term<br>Answer 0.2616 or better or 0.262 only |
| | (ii) | Binomial requires being chosen independently, which this is not, but unimportant as population is large | B2 **2** | Focus on "Without replacement" negating independence condition. It doesn't negate "constant probability" condition but can allow B1 if "selected". See notes |
| **3** | (i) | $\left(\dfrac{32-40}{\sigma}\right) = \Phi^{-1}(0.2) = -0.842$<br>$\sigma = 9.5[06]$ | M1<br>B1<br>A1 **3** | Standardise and equate to $\Phi^{-1}$, allow "1 –" errors, $\sigma^2$, cc<br>0.842 seen<br>Answer, 9.5 or in range [9.50, 9.51], c.w.o. |
| | (ii) | B(90, 0.2)<br>$\approx$ N(18, 14.4)<br>$1-\Phi\left(\dfrac{19.5-18}{\sqrt{14.4}}\right) = 1 - \Phi(0.3953)$<br>$= 1 - 0.6537 \quad = \mathbf{0.3463}$ | B1<br>M1<br>A1<br>M1<br>A1<br>A1 **6** | B(90, 0.2) stated or implied<br>N, their $np$ …<br>… variance their $npq$, allow √ errors<br>Standardise with $np$ and $npq$, allow √, cc errors, e.g. .396, .448, .458, .486, .472; √$npq$ and cc correct<br>Answer, a.r.t. 0.346 [NB: 0.3491 from Po: 1/6] |
| **4** | | $H_0: p = 0.4$,<br>$H_1: p > 0.4$<br>$R \sim B(16, 0.4)$: | B1<br>B1<br>M1 | Fully correct, B2. Allow $\pi$. $p$ omitted or $\mu$ used in both, or > wrong: B1 only. $x$ or $\bar{x}$ or 6.4 etc: B0<br>B(16, 0.4) stated or implied, allow N(6.4, 3.84) |
| | (α) | $P(R \ge 11) = 0.0191$<br>$> 0.01$ | A1<br>A1 | Allow for $P(\le 10) = 0.9808$, and $< 0.99$, or $z = 2.092$ or $p = 0.018$, but *not* $P(\le 11) = 0.9951$ or $P(= 11) = 0.0143$<br>Explicit comp with .01, or $z$<2.326, *not* from ≤11 or =11 |
| | (β) | CR $R \ge 12$ and $11 < 12$<br>Probability 0.0049 | A1<br>A1 | Must be clear that it's ≥ 12 and not ≤ 11<br>Needs to be seen, allow 0.9951 here, or $p$ = .0047 from N |
| | | Do not reject $H_0$. Insufficient evidence that proportion of commuters who travel by train has increased | M1<br>A1 FT **7** | Needs like-with-like, $P(R \ge 11)$ or CR $R \ge 12$<br>Conclusion correct on their $p$ or CR, contextualised, not too assertive, e.g. "evidence that" needed.<br>Normal, $z$ = 2.34, "reject" [no cc] can get 6/7 |
| **5** | (i) | (a) $30 + 1.645 \times \dfrac{5}{\sqrt{10}}$<br>$= 32.6$<br>Therefore critical region is $\bar{t} > 32.6$ | M1<br>B1<br>A1<br>A1 FT **4** | $30 + 5z/\sqrt{10}$, allow ± but not just –, allow √ errors<br>$z$ = 1.645 seen, allow –<br>Critical value, art 32.6<br>"$> c$" or "$\ge c$", FT on $c$ provided > 30, can't be recovered. Withhold if not clear which is CR |
| | | (b) $P(\bar{t} < 32.6 \mid \mu = 35)$<br>$\dfrac{32.6-35}{5/\sqrt{10}}$ [= –1.5178]<br>**0.0645** | M1*<br>dep*M1<br>A1 **3** | Need their $c$, final answer < 0.5 and $\mu$ = 35 at least, but allow answer > 0.5 if consistent with their (i)<br>Standardise their CV with 35 and √10 or 10<br>Answer in range [0.064, 0.065], or 0.115 from 1.96 in (a) |
| | (ii) | $(32.6 - \mu) = 0$<br>$\mu = 32.6$<br>$20 + 0.6m = 32.6$<br>$m = \mathbf{21}$ | M1<br>A1 FT<br>M1<br>A1 **4** | Standardise $c$ with $\mu$, equate to $\Phi^{-1}$, can be implied by: $\mu$ = their $c$<br>Equate and solve for $m$, allow from 30 or 35<br>Answer, a.r.t. 21, c.a.o.<br>MR: 0.05: M1 A0 M1, 16.7 A1 FT<br>Ignore variance throughout (ii) |

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| **6** | (a) | N(24, 24)<br>$1-\Phi\left(\frac{30.5-24}{\sqrt{24}}\right) = 1 - \Phi(1.327)$<br>$= \mathbf{0.0923}$ | B1<br>B1<br>M1<br>A1<br>A1 **5** | Normal, mean 24 stated or implied<br>Variance or SD equal to mean<br>Standardise 30 with $\lambda$ and $\sqrt{\lambda}$, allow cc or $\sqrt{}$ errors, e.g. .131 or .1103 ; 30.5 and $\sqrt{\lambda}$ correct<br>Answer in range [0.092, 0.0925] |
| | (b)(i) | $p$ or $np$ [= 196] is too large | B1 **1** | Correct reason, no wrong reason, don't worry about 5 or 15 |
| | (ii) | Consider $(200 - E)$<br>$(200 - E) \sim \text{Po}(4)$<br>$\text{P}(\geq 6)$ [= 1 – 0.7851]<br>$= \mathbf{0.2149}$ | M1<br>M1<br>M1<br>A1 **4** | Consider complement<br>Po(200×0.02)<br>Poisson tables used, correct tail, e.g. 0.3712 or 0.1107<br>Answer a.r.t. 0.215 only |
| **7** | | $H_0 : \mu = 56.8$<br>$H_1 : \mu \neq 56.8$<br>$\bar{x} = 17085/300 = 56.95$<br>$\frac{300}{299}\left(\frac{973847}{300} - 56.95^2\right)$<br>$= 2.8637\ldots$ | B2<br><br>B1<br>M1<br>M1<br>A1 | Both correct<br>One error: B1, but *not* $\bar{x}$, etc<br>56.95 or 57.0 seen or implied<br>Biased [2.8541] : M1M0A0<br>Unbiased estimate method, allow if ÷ 299 seen anywhere<br>Estimate, a.r.t. 2.86 [not 2.85] |
| | (α) | $z = \frac{56.95 - 56.8}{\sqrt{2.8637/300}} = 1.535$<br>$1.535 < 1.645$ or $0.0624 > 0.05$ | M1<br>A1<br>A1 | Standardise with $\sqrt{300}$, allow $\sqrt{}$ errors, cc<br>$z \in [1.53, 1.54]$ or $p \in [0.062, 0.063]$, *not* – 1.535<br>Compare explicitly $z$ with 1.645 or $p$ with 0.05, or $2p > 0.1$, not from $\mu = 56.95$ |
| | (β) | CV $56.8 \pm 1.645 \times \sqrt{\frac{2.8637}{300}}$<br>$56.96 > 56.95$ | M1<br>A1<br>A1 FT | $56.8 + z\sigma/\sqrt{300}$, needn't have ±, allow $\sqrt{}$ errors<br>$z = 1.645$<br>$c = 56.96$, FT on $z$, and compare 56.95 [$c_L = 56.64$] |
| | | Do not reject $H_0$;<br>insufficient evidence that mean thickness is wrong | M1<br>A1 FT **11** | Consistent first conclusion, needs 300, correct method and comparison<br>Conclusion stated in context, not too assertive, e.g. "evidence that" needed |
| **8** | (i) | $\int_1^\infty kx^{-a}dx = \left[k\frac{x^{-a+1}}{-a+1}\right]_1^\infty$<br>Correctly obtain $k = a - 1$ **AG** | M1<br>B1<br>A1 **3** | Integrate f($x$), limits 1 and ∞ (at some stage)<br>Correct indefinite integral<br>Correctly obtain given answer, don't need to see treatment of ∞ but mustn't be wrong. Not $k^{-a+1}$ |
| | (ii) | $\int_1^\infty 3x^{-3}dx = \left[3\frac{x^{-2}}{-2}\right]_1^\infty = 1\frac{1}{2}$<br>$\int_1^\infty 3x^{-2}dx = \left[3\frac{x^{-1}}{-1}\right]_1^\infty - (1\frac{1}{2})^2$<br>Answer ¾ | M1<br><br>M1<br>A1<br>M1<br>A1 **5** | Integrate $x$f($x$), limits 1 and ∞ (at some stage) [$x^4$ is *not* MR]<br>Integrate $x^2$f($x$), correct limits<br>Either $\mu = 1\frac{1}{2}$ or E($X^2$) = 3 stated or implied, allow $k$, $k/2$<br>Subtract their numerical $\mu^2$, allow letter if subs later<br>Final answer ¾ or 0.75 only, cwo, e.g. not from $\mu = -1\frac{1}{2}$.<br>[SR: Limits 0, 1: can get (i) B1, (ii) M1M1M1] |
| | (iii) | $\int_1^2 (a-1)x^{-a}dx = \left[-x^{-a+1}\right]_1^2 = 0.9$<br>$1 - \frac{1}{2^{a-1}} = 0.9,\ 2^{a-1} = 10$<br>$a = 4.322$ | M1*<br><br>dep*M1<br>M1 indept<br>A1 **4** | Equate ∫f($x$)d$x$, one limit 2, to 0.9 or 0.1.<br>[Normal: 0 ex 4]<br>Solve equation of this form to get $2^{a-1}$ = number<br>Use logs or equivalent to solve $2^{a-1}$ = number<br>Answer, a.r.t. 4.32. T&I: (M1M1) B2 or B0 |

***Specimen Verbal Answers***

**1** α "Cases of infection must occur randomly, independently, singly and at constant average rate" B0

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|---|---|---|---|
| **1** | α | "Cases of infection must occur randomly, independently, singly and at constant average rate" | B0 |
| | β | Above + "but it is contagious" | B1 |
| | γ | Above + "but not independent as it is contagious" | B2 |
| | δ | "Not independent as it is contagious" | B2 |
| | ε | "Not constant average rate", or "not independent" | B0 |
| | λ | "Not constant average rate because contagious" *[needs more]* | B1 |
| | ζ | "Not constant average rate because more likely at certain times of year" | B2 |
| | μ | Probabilities changes because of different susceptibilities | B0 |
| | ν | Not constant average rate because of different susceptibilities | B2 |
| | η | Correct but with unjustified or wrong extra assertion *[scattergun]* | B1 |
| | θ | More than one correct assertion, all justified | B2 |
| | π | Valid reason (e.g. "contagious") but not referred to conditions | B1 |

*[Focus is on explaining why the required assumptions might not apply. No credit for regurgitating learnt phrases, such as "events must occur randomly, independently, singly and at constant average rate, even if contextualised.]*

**2** Don't need either "yes" or "no".

| | | | |
|---|---|---|---|
| | α | "No it doesn't invalidate the calculation" *[no reason]* | B0 |
| | β | "Binomial requires not chosen twice" *[false]* | B0 |
| | γ | "Probability has to be constant but here the probabilities change" | B0 |
| | δ | Same but "probability <u>of being chosen</u>" *[false, but allow B1]* | B1 |
| | ε | "Needs to be independently chosen but probabilities change" *[confusion]* | B0 |
| | ζ | "Needs to be independent but one choice affects another" *[correct]* | B2 |
| | η | "The sample is large so it makes little difference" *[false]* | B0 |
| | θ | "The population is large so it makes little difference" *[true]* | B2 |
| | λ | Both correct and wrong reasons (scattergun approach) | B1 |

*[Focus is on modelling conditions for binomial: On every choice of a member of the sample, each member of the population is equally likely to be chosen; and each choice is independent of all other choices.*
*Recall that in fact even without replacement the probability that any one person is chosen is the same for each choice. Also, the binomial "independence" condition <u>does</u> require the possibility of the same person being chosen twice.]*

Some explanation seems necessary. The following are widespread but mistaken beliefs:
1) Choosing a random sample by means of random numbers does not permit the same person to be chosen twice.
2) Sampling without replacement causes $p$ to change from one trial to another.

Both of these are *FALSE*! Why?
1) Random sampling using random numbers demands that each member of the sample is chosen independently of every other member of the sample. If it is known that a certain person is in the sample and that that person cannot be chosen again, this fact changes the probability that another person is chosen next time. The same sequence of random digits can come up again. Just because, say, 123 has already occurred doesn't alter the fact that 123 is just as likely as any other 3-digit sequence to come up on any other go, and the same person can be chosen twice.
2) Attention has been drawn before to the confusion that exists for many candidates between "trials are independent" and "each trial has the same probability of success", caused by too much emphasis on the misleading example of drawing counters out of a bag. Consider the present case. The probability that, say, the third student picked is a science student is 0.35, as it is for the first, second, …, tenth. This is a familiar fact from S1 and can easily be demonstrated using a tree diagram, assuming an appropriate total

population size (say 100). It is not the absolute ("prior") probabilities that change but the *conditional* probabilities, which are irrelevant.

In fact the binomial distribution applies only to sampling *with* replacement. Strictly, the proper method of calculating probabilities when sampling *without* replacement is the method using ${}^nC_r$ from S1. Again suppose the population is of size 100, of whom 35 are studying science subjects. Consider the probability that a sample of 10 students consists of exactly two who are studying science subjects.

- Case 1 (with replacement. Binomial): ${}^{10}C_2\ 0.35^2\ 0.65^8 = 0.1757$.
- Case 2 (without replacement. ${}^nC_r$): ${}^{35}C_2 \times {}^{65}C_8 / {}^{100}C_{10} = 0.1735$.

The difference is small, though not non-existent. The bigger the population, the smaller the difference; for a population of size 1000 the second probability is 0.1755. In real life, repeats are usually not allowed, but use of the binomial distribution remains appropriate provided the population is large enough. (There is a technical name for the ${}^nC_r$ method; it is called the *hypergeometric distribution*.)